Visualizing the Isodirection Lines of a Vector Field

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RNR Technical Report RNR-92-017, March, 1992

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Abstract

For a vector field in $\mathbf{R}^3$, $V(x, y, z)$, *and a normalized direction vector,* (n_1, n_2, n_3), *an isodirection line is defined as the set of points satisfying* $V(x, y, z) = (mn_1, mn_2, mn_3)$, *where* $m \geq 0$. *An algorithm is presented for constructing the isodirection line for a given direction over two- and three-dimensional vector fields sampled over a curvilinear grid. An array of isodirection lines constructed through a vector field provides a visual image which captures the topology of the vector field. Critical points of the vector field, where the magnitude of the vector vanishes, are located as a consequence of the isodirection line construction. Isodirection lines can also guide the placement of the initial points for generating integral curves which add to the ability of the image to characterize the topology of the vector field.*

1. Introduction

Scientific visualization utilizes the pattern-recognition capabilities of the visual sense to analyze much greater quantities of data than is possible with purely numeric approaches [2]. The raw numeric data often is in the form of samples at discrete locations arranged in a regular or irregular pattern. The data commonly represents a scalar field, for which there is a single value for each sample point, or a vector field, for which there is a vector for each sample point.

The methods used to visualize scalar fields are reasonably successful in displaying the structure of the field. A range of colors can be used to represent a range of scalar values. Whole two-dimensional scalar fields or surface subsets of three-dimensional fields can be so colored to present an image in which regions of particular scalar values can easily be distinguished from other regions.

Isoscalar contour lines are very useful for mapping the structure of two-dimensional scalar fields. The utility of topographic maps for displaying elevation information with elevation contour lines, for instance, is widely appreciated. Isoscalar surfaces are used in a number of scientific disciplines to display the structure of three-dimensional scalar fields.

Various visualization methods are employed to gain an understanding of the structure or topology of sampled vector fields. They range from simply drawing directed line segments, representing the direction and magnitude of the vector at each sampled point, to the exploration of integral manifolds by location and analysis of the critical points in the vector field [4].

An *isodirection line* is a line or curve which connects vectors in the vector field which are of the same direction. The visual image of a discrete number of isodirection lines through a vector field maps the structure of the vector field much in the same way that isoscalar contour lines or isoscalar surfaces map the structure of a scalar field.

A method for constructing isodirection lines from vector fields sampled at the vertices of two- and three-dimensional curvilinear grids is presented here. A method for the location of the critical points of a vector field, where the magnitude of the vector vanishes, is a natural consequence of isodirection line construction and is also presented. The example data is drawn from the study of computational fluid dynamics (CFD).

2. Background

A variety of techniques have been used to visualize the structure of sampled vector fields common to the study of CFD [1, 4, 11 16]. Chief among them has been the use of integral curves or surfaces, which are everywhere tangent to interpolated vector samples [3, 4, 7, 8, 16]. The algorithms for the construction of integral curves or surfaces through a sampled vector field solve initial value problems, where an initial seed location in the vector field is designated and subsequent locations are found by numerical integration. While these techniques can be effective, caution must be used in the selection of step size and numerical integration method lest erroneous results occur [14].

The effectiveness in showing the overall structure of the vector field with integral curves or surfaces is dependant upon the location of the seed points. Interactive placement of seed points with quick display of the resulting trace is one way of constructing a useful image [1, 8].

Techniques for locating the critical points of vector fields have been combined with visualizing integral curves and surfaces to depict the topology of vector fields [4, 5, 6, 7]. Critical point location and analysis is helpful in guiding the selection of locations to begin integral curves and surfaces. Integral curves initiated near critical points trace paths which depict important features of the flow [4, 5].

A method used to sketch approximate integral curves of analytic functions is the method of *isoclines* [19]. Given a differential equation:

$$\frac{dx}{dy} = f(x, y)$$

an isocline is a curve on which the slope $f(x, y)$ of the vector field has a constant value, c. Figure 1 shows a set of isoclines drawn with line segments indicating the slope the isocline represents. Three integral curves are sketched everywhere tangent to the inclination.

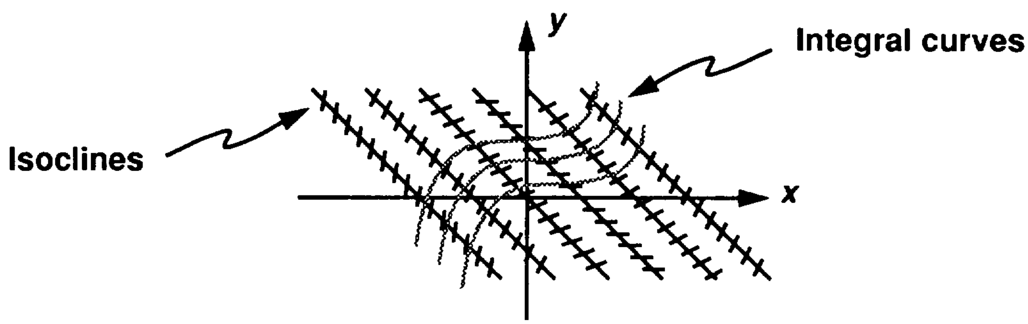


Figure 1: Isoclines used to sketch integral curves.

The use of isoclines to sketch out integral curves for analytic functions is indicative of the usefulness of the isoclines to depict the basic structure of the analytic function. A similar structure, termed an *isotangent curve*, has been used for computing two-dimensional phase portraits for oriented textures[20].

The ability of isoclines to portray the structure of analytic functions in two dimensions provides motivation for developing an algorithm for constructing similar structures in sampled vector fields of two and three dimensions. An isodirection line is just such a structure with direction rather than inclination as the characteristic of interest. Two (opposite) directions are assimilated in an inclination.

An isoscalar surface of a three-dimensional scalar field is the set of points satisfying $f(x, y) = c$ for some constant value, c. The marching cubes algorithm [13] and its variations [3, 11, 15, 22, 23] are probably the most widely used algorithms for constructing isoscalar surfaces. One important feature of this algorithm is the divide and conquer approach it takes, constructing surface fragments within one cubical cell at a time. The collection of fragments over the entire grid forms the isoscalar surface. This allows for the use of an arbitrary progression through the data; the processing of subsequent cells is not dependant on the processing of prior cells. Isodirection line construction employs a similar divide and conquer approach using a subdivision of the curvilinear grid into triangles for two-dimensional grids and tetrahedra for three-dimensional grids. Isodirection line segments are constructed for each triangle or tetrahedra. A collection of these segments forms the complete isodirection line.

3. Two-dimensional Isodirection Line Construction

Construction of isodirection lines in a sampled vector field is similar to the construction of isoscalar contour lines in a sampled scalar field. Before we discuss the construction of isodirection lines, let us examine an algorithm for construction of isoscalar contours. Given a scalar field sampled over a curvilinear grid and a scalar value, the steps for construction of a isoscalar contour are:

Step 1: Divide the curvilinear grid into triangular cells.
Step 2: Test if the desired scalar value is between the scalar values at the vertices of each edge of each cell.
Step 3: If the desired scalar value is between the scalar values of the vertices of a cell edge, use linear interpolation to find the location on the cell edge which is of the desired scalar value. This location is an edge - contour line intersection point.
Step 4: Draw a line between the edge - contour line intersections found in each cell.

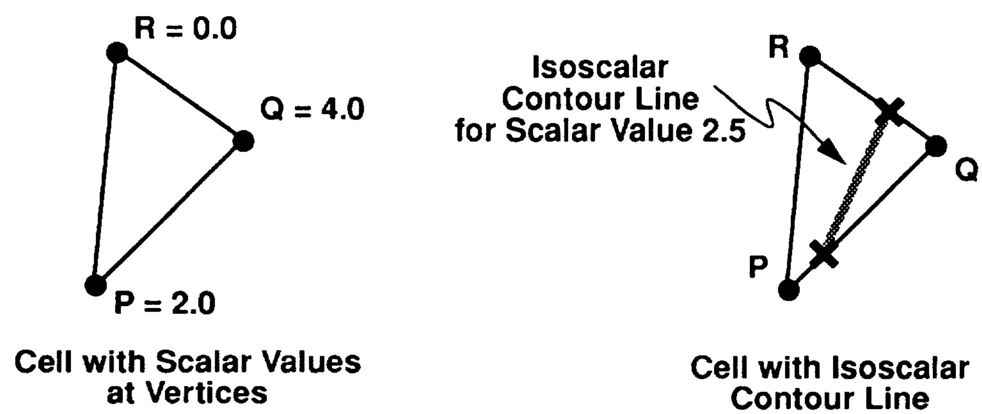


Figure 2: Two-dimensional isoscalar contour line construction.

An example is illustrated in figure 2. A desired scalar value, 2.5, is found to be between the scalar values for two edges of the triangular cell **PQR** with scalar values 2.0, 4.0, and 0.0, respectively. The location for 2.5 along the edge between **P** and **Q** is found using linear interpolation. The resulting location corresponds to a cell edge - contour line intersection point. An edge - contour line intersection is also located on the edge between **R** and **Q**. Finally, the contour line is drawn connecting the two edge - contour line intersections.

For a vector field in $\mathbf{R}^2$, $V(x, y)$, and a normalized direction vector, (n_1, n_2), an isodirection line is defined as the set of points satisfying $V(x, y) = (mn_1, mn_2)$, where $m \geq 0$.

The construction of an isodirection line follows a pattern similar to the construction of an isoscalar contour. Given a vector field sampled over a two-dimensional curvilinear grid and a direction vector, the steps for construction of a isodirection line are:

Step 1: Divide the curvilinear grid into triangular cells.
Step 2: Test if the desired direction is between the directions of the vectors at the vertices of each edge of each cell.
Step 3: If the desired direction is between the directions of the vectors at the vertices of a cell edge, use linear interpolation to find the location on the cell edge which is of the desired direction. This location is an edge - isodirection line intersection point.
Step 4: Draw a line between the edge - isodirection line intersections found in each cell.

One "betweeness" test for direction in step 2 translates the vectors at the vertices of a cell edge to a common tail with the desired direction vector. An order is determined where the vectors from the vertices of the cell edge form an angle of less than 180 degrees. If the desired direction is between the two given vectors then it will intersect a line drawn between the heads of the two vectors.

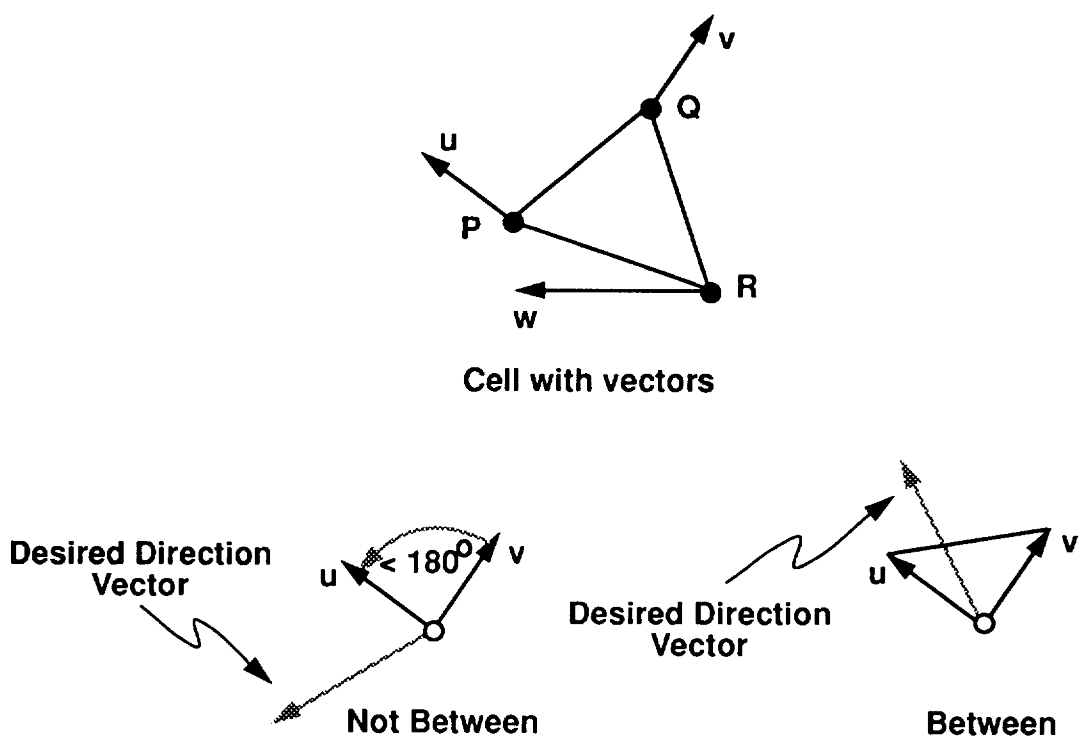


Figure 3: Betweeness test for edge **PQ**.

A line between the heads of the two vectors **u** and **v** transposed to a common tail is used in a way analogous to the number line in the linear interpolation for scalar values in figure 3. The intersection point between the line and the desired direction vector is used to determine the ratio of distances between **P**, **Q** and a point with the desired direction vector value (figure 4).

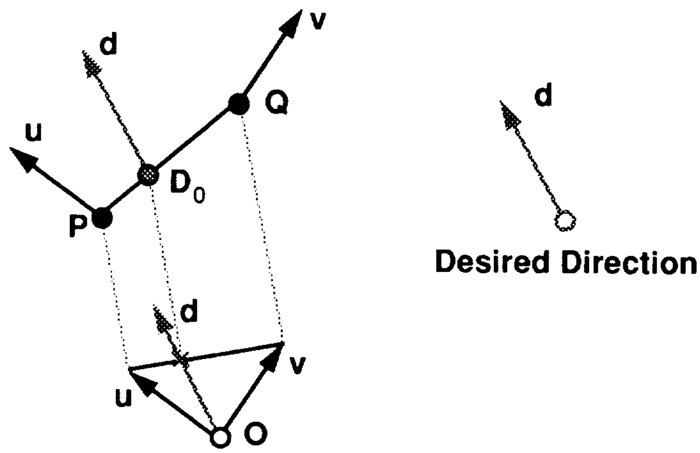


Figure 4: Interpolation to find cell edge - isodirection line intersection point.

An piecewise-linear approximation of an isodirection line for a particular direction in a two-dimensional vector field can be drawn connecting the cell - isodirection line intersection points. In figure 5, a triangular cell with vectors at points **P**, **Q**, and **R** is given. The betweeness test is used on the vector pairs for each edge of the triangular cell. In this example, the desired direction vector, **d**, lies between **v** - **w** and **u** - **w**. This means that the isodirection line for **d** intersects the triangular cell in two locations. The intersecting locations are found on the edges by interpolation and an approximate isodirection line is drawn.

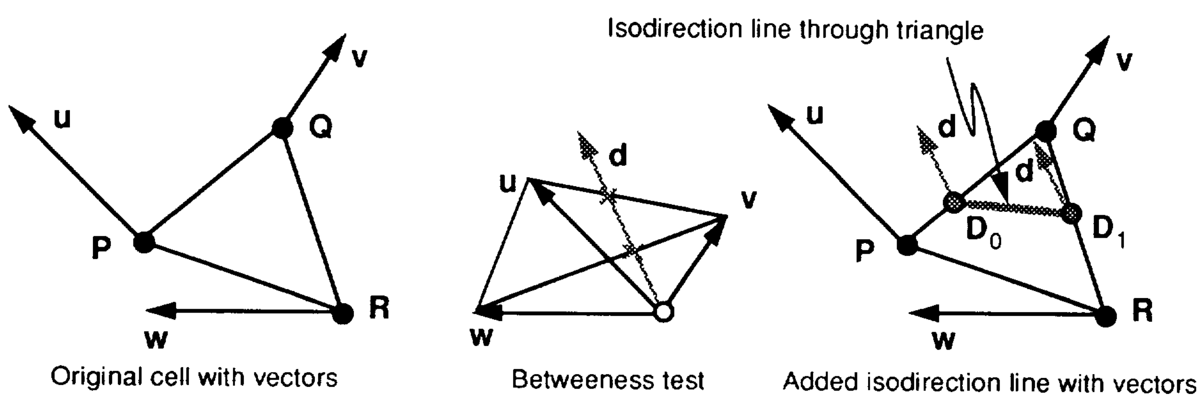
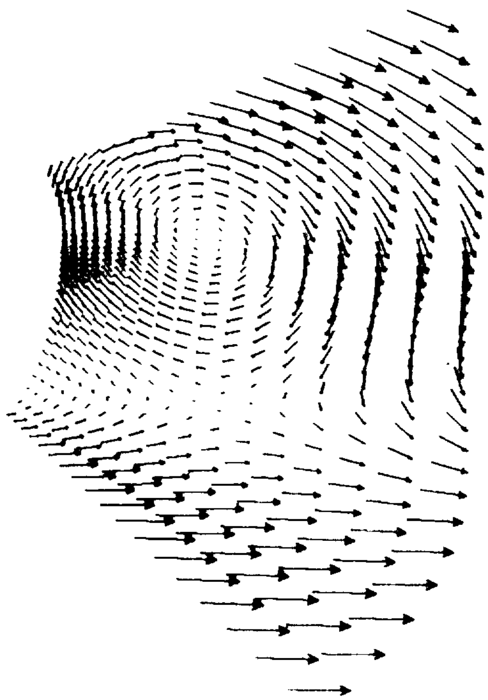


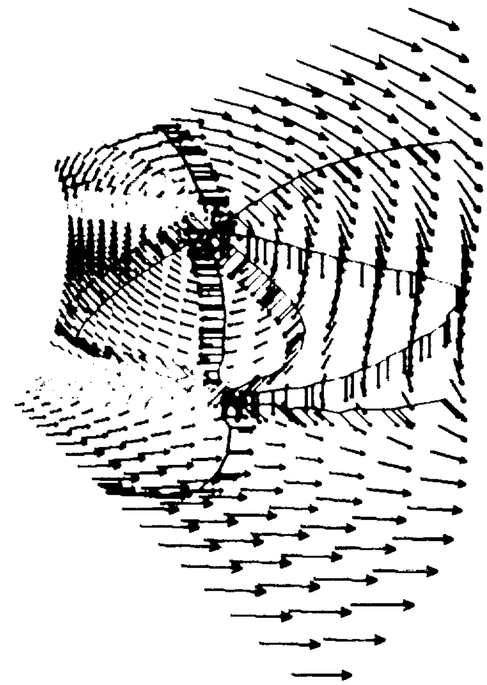
Figure 5: Isodirection line construction on triangular cell.

As in the algorithm for construction isoscalar contour lines, a vector field sampled at the vertices of a curvilinear grid is divided into triangular cells for isodirection line construction. Since interpolation along the common edge of two adjacent triangular cells will result in the same location, the line segments will align to form a curve which spans multiple cells.

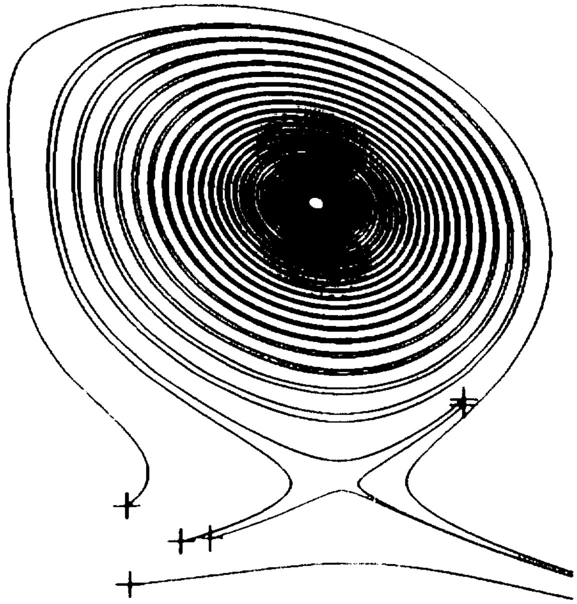
Figure 6 shows various visual representations of a two-dimensional subset of the



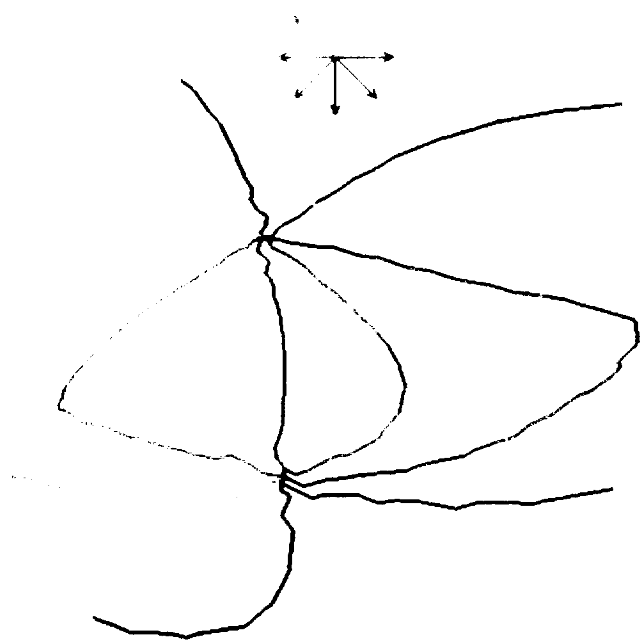
a: Vectors displayed as arrows on subset of data (540 nodes).



b: Isodirection lines with vectors displayed as arrows



c: Integral curves with inital points at cross hairs.



d: Isodirection lines with direction icon.

Figure 6: Subset of two dimensional flow about a cylinder.

data produced by the numerical simulation of flow past a tapered cylinder [10]. Figure 6a displays the vectors as arrows with the tail end of the arrow on the sampled vertex of the curvilinear grid. The arrows are oriented in the direction and proportional to the magnitude of the sampled vector.

In figure 6b isodirection lines are added to the sampled vectors together with the arrows indicating the direction the isodirection line represents. The lines are color coded by direction. Six integral curves are drawn over the same subset of the data in figure 6c. Figure 6d displays the isodirection lines only.

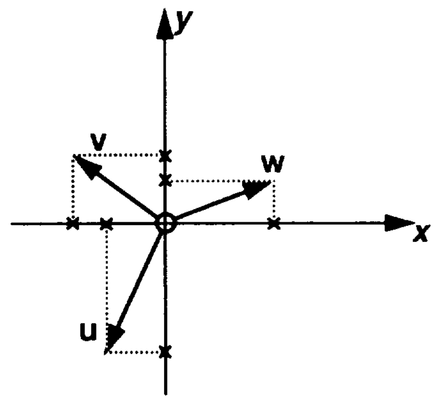
4. Two-Dimensional Critical Point Location

A critical point is a location in the vector field where the magnitude of the vector vanishes. Each component of the vector at the critical point equals zero. One way to approximate the position of a critical point in two dimensions is to find the location where isoscalar contour lines for $v_x = 0$ and $v_y = 0$ intersect.

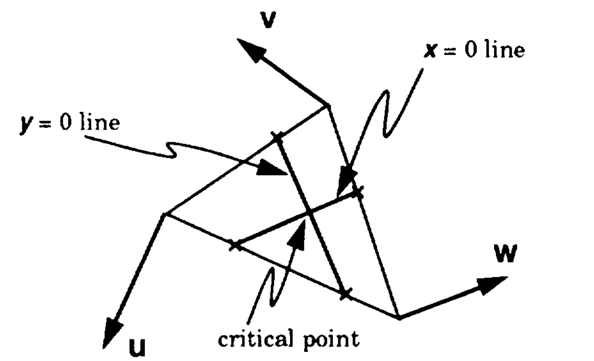
- Step 1: Divide the curvilinear grid into triangular cells.
- Step 2: Test if the cell contains a critical point.
- Step 3: If the cell contains a critical point, find isoscalar contour lines for $v_x = 0$ and $v_y = 0$ for the cell.
- Step 4: Solve the system of linear equations for the two isoscalar contour lines to find the position of the critical point.

Figure 7a-b shows an example of the location of a critical point within a triangular cell. Figure 7a shows the vectors at the vertices of the cell projected to the ***x*** and ***y*** axes. These projections are used to find the intersections of the cell edge and the isoscalar contour lines for $v_x = 0$ and $v_y = 0$. The intersection of the two contour lines at the critical point is shown in figure 7b.

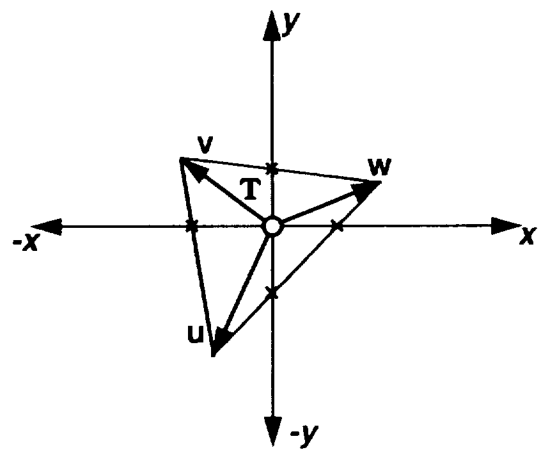
Figure 7c-d shows the construction of isodirection lines for four compass directions, ***x***, ***-x***, ***y***, and ***-y***. Figure 7c shows the interpolation to find the cell edge - isodirection line intersection points for the four directions (see figure 4). Isodirection lines are defined as including points where the magnitude of the vector is zero (critical points). The four isodirection lines are drawn in figure 7d showing the connection of the edge-isodirection line intersections and the critical point. Vectors are drawn at the edge-isodirection line intersection point to indicate the direction the isodirection line represents.



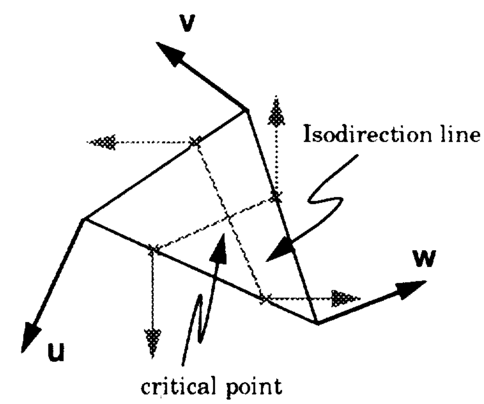
a: Projection to x and y axes.



b: Location of critical point.



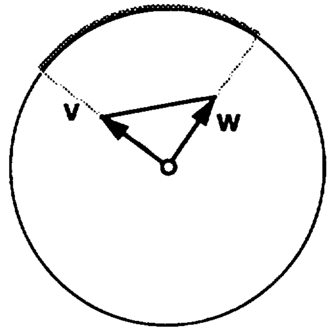
c: Vector interpolation for directions x, $-x$, y, and $-y$.



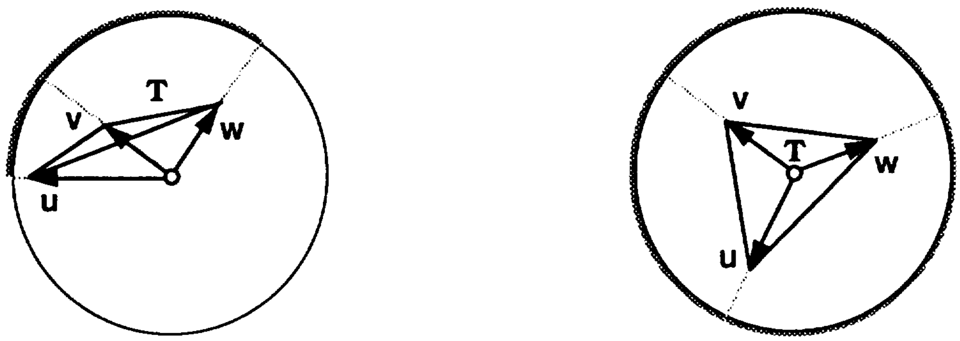
d: Critical point and isodirection lines for x, $-x$, y, and $-y$.

Figure 7: Locating a two-dimensional critical point.

It was shown that for a direction vector to be between two given vectors it must intersect a line drawn between the heads of the two given vectors (figure 3). If a circle represents all possible two-dimensional directions, the gray arc on the circle in figure 8a represents those directions which are between **v** and **w**. The directions of a triangular cell are mapped to three overlapping arcs on the circle. These three arcs double cover the arc between **u** and **w**. The heads of the three vectors of the triangular cell transposed to a common tail form a small triangle (call it **T**). An intersection between **T** and a direction vector corresponds to the isodirection line intersection with the triangular cell (see figure 5). For a configuration such as in figure 8b, any direction will intersect **T** in two places or not at all (except at the end vertices). This means that if all three vectors lie in the same semicircle, the isodirection line passes through the cell.



a: Directions between **v** and **w** mapped to circle.



b: **T** does not contain common tail. No critical point is present.

c: **T** contains common tail. There is a critical point in the cell.

Figure 8: Test for critical point in cell.

A special case is when **T** contains the common tail or equivalently, when the three given vectors do not lie in the same semicircle (figure 8c). For this special case an arbitrary direction vector will intersect **T** at one and only one location since **T** spans all directions. This special case is indicative of the triangular cell containing a critical point, assuming monotonic interpolation schemes are applied for values in the cell interior. A simple test to determine whether the three vectors of a cell lie in a semicircle also determines the presence of a critical point.

Referring back to figure 6, note the presence of two critical points where the isodirection lines converge in figure 6d. A vortex center and a saddle center is illustrated by the integral curves in figure 6c. The positions of the vortex center and saddle center are in the same positions as the critical points located with the isodirection lines in figure 6d.

5. Three-dimensional Isodirection Line Construction

For a vector field in $\mathbf{R}^3$, $V(x, y, z)$, and a normalized direction vector, (n_1, n_2, n_3), an isodirection line is defined as the set of points satisfying $V(x, y, z) = (mn_1, mn_2, mn_3)$, where $m \geq 0$.

The construction of an isodirection line in three dimensions is similar to its construction in two dimensions. For three dimensions, intersection points of the isodirection line with the faces of a tetrahedral cell are found rather than with a triangular cell. Given a vector field sampled over a three-dimensional curvilinear grid and a direction vector, for construction of an isodirection line:

- Step 1: Divide the curvilinear grid into tetrahedral cells.
- Step 2: Test if the desired direction is between the directions of the vectors at the vertices of each face of each tetrahedral cell.
- Step 3: If the desired direction is between the directions of the vectors at the vertices of a cell face, use interpolation to find the location on the cell face which is of the desired direction. This location is a face - isodirection line intersection point.
- Step 4: Draw a line between the face - isodirection line intersections found in each cell.

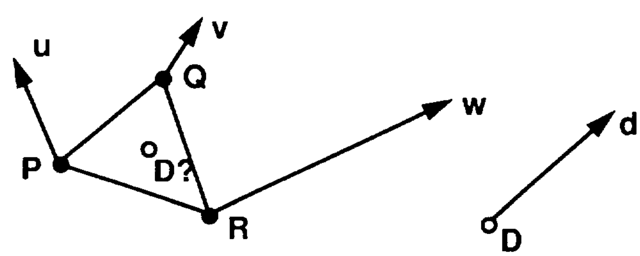


Figure 9: Is a vector in the direction of **d** on this triangular face of a tetrahedral cell?

In step 2 a betweeness test is applied to determine whether the desired direction vector, **d**, lies between the vectors at the vertices of a tetrahedral face, **u**, **v**, and **w** (figure 9). The vectors are transposed to a common tail. A vector in the desired direction lies in the tetrahedral face if **d** intersects a triangle formed by the heads of **u**, **v**, and **w** (figure 10a).

To find out if d intersects a triangle formed by the heads of **u**, **v**, and **w** a series of tests are performed to determine is the head of d is corralled by the planes formed by pairs of given vectors. In order for **d** to be between **u**, **v**, and **w**, the head of **d** and the head of one given vector must be on the same side of the plane formed by the two other given vectors. In figure 10b, after transposing the vectors to a common tail, a plane is formed by vectors **u** and **v**. The heads of both the third given vector, **w**, and the desired direction vector, **d**, are on the same side of the plane **uv**.

a: **d** between vectors **u**, **v**, and **w**.

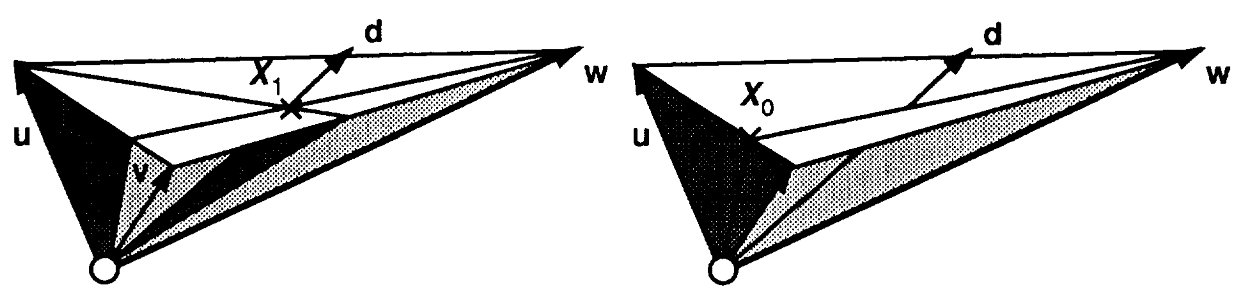
b: **d** and **w** on same side of plane **uv**.

Figure 10: Betweeness test.

Two more tests in this example would be made to show that the heads of **v** and **d** are on the same side of the plane formed by **u** and **w**, and that the heads of **u** and **d** are on the same side of the plane formed by **v** and **w**. If all three tests are true then **d** will intersect a triangle formed by the heads of **u**, **v**, and **w**.

Once it has been determined that the desired vector lies between the vectors associated with the vertices of the face of the tetrahedra, it remains to find the location of the vector on the face that is in the desired direction. The location of the intersection between the desired direction vector and the triangle formed by the heads of **u**, **v**, and **w** as in the above diagram is used with bilinear interpolation to locate a point on the face of the tetrahedra.

Two points are used for the interpolation, X_0 and X_1, which are each found by finding the intersection of three planes. In figure 11a the three planes whose intersection is at X_1 are the planes formed by **d** and **w**, **d** and **u** and the heads of **u**, **v**, and **w**. In figure 11b the three planes whose intersection is at X_0 are the planes formed by **d** and **w**, **u** and **v**, and the heads of **u**, **v**, and **w**.



a: Three planes intersecting at X_1.

b: Three planes intersecting at X_0.

Figure 11: Intersection of planes.

The plane intersection points are determined by solving the system of linear equations for the three planes. The ratio of the distance from the head of **u** to X_0 to the distance from the head of **u** to the head of **v** is used to linearly interpolate along the **P-Q** edge of the tetrahedral face (call it x_0). The ratio of the distance from X_0 to X_1 to the distance from X_0 to the head of **w** is used to linearly interpolate along the line from **R** to $\boldsymbol{x}_0$ (figure 12).

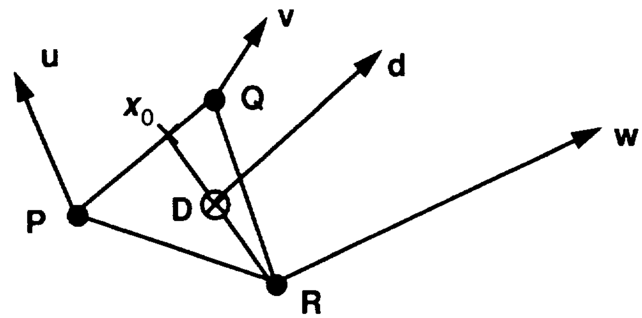


Figure 12: Location of vector in the direction of **d** on tetrahedral face.

All four faces of the tetrahedra are tested for intersection with the isodirection line as above. When the isodirection line intersects the tetrahedra, the usual case is to find two faces with intersection points. A line drawn between these two points is the approximate isodirection line through the tetrahedra (figure 13).

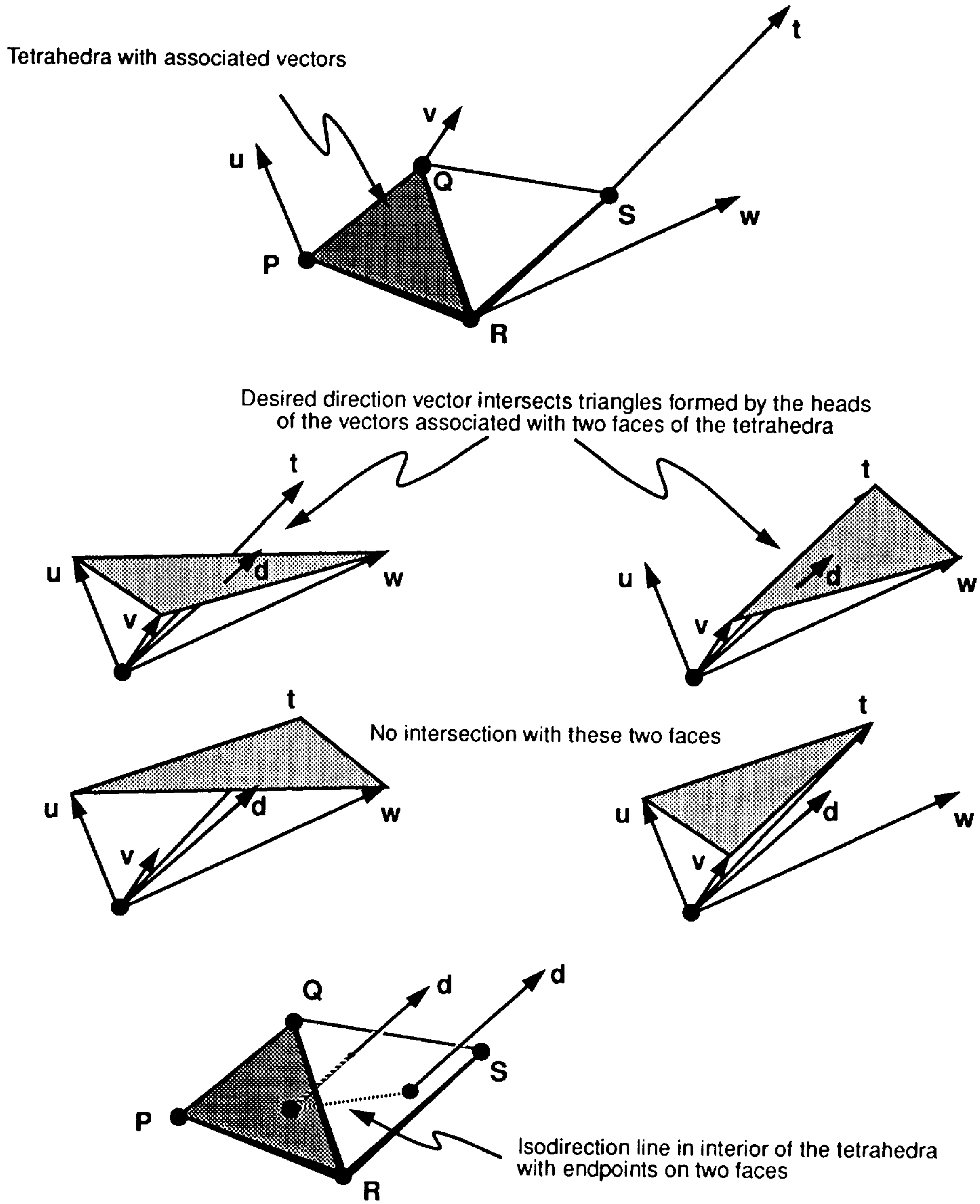


Figure 13: Isodirection line for **d** constructed for tetrahedral cell.

6. Three-Dimensional Critical Point Location

Each component of the critical point vector is equal to zero. One way of locating the approximate position of a critical point in three dimensions is to find the location where isoscalar surfaces for $v_x = 0$, $v_y = 0$, and $v_z = 0$ intersect. Given a vector field sampled over a three-dimensional curvilinear grid, for critical point location:

- Step 1: Divide the curvilinear grid into tetrahedral cells.
- Step 2: Test if the cell contains a critical point.
- Step 3: If the cell contains a critical point, find isoscalar surfaces for $v_x = 0$, $v_y = 0$, and $v_z = 0$ for the cell.
- Step 4: Solve the system of linear equations for the three isoscalar surfaces to find the position of the critical point.

Figure 14 shows an example of the location of a critical point within a tetrahedral cell. The planar isosurfaces for $v_x = 0$, $v_y = 0$, and $v_z = 0$ are found using a variation of the marching cubes algorithm for tetrahedra.

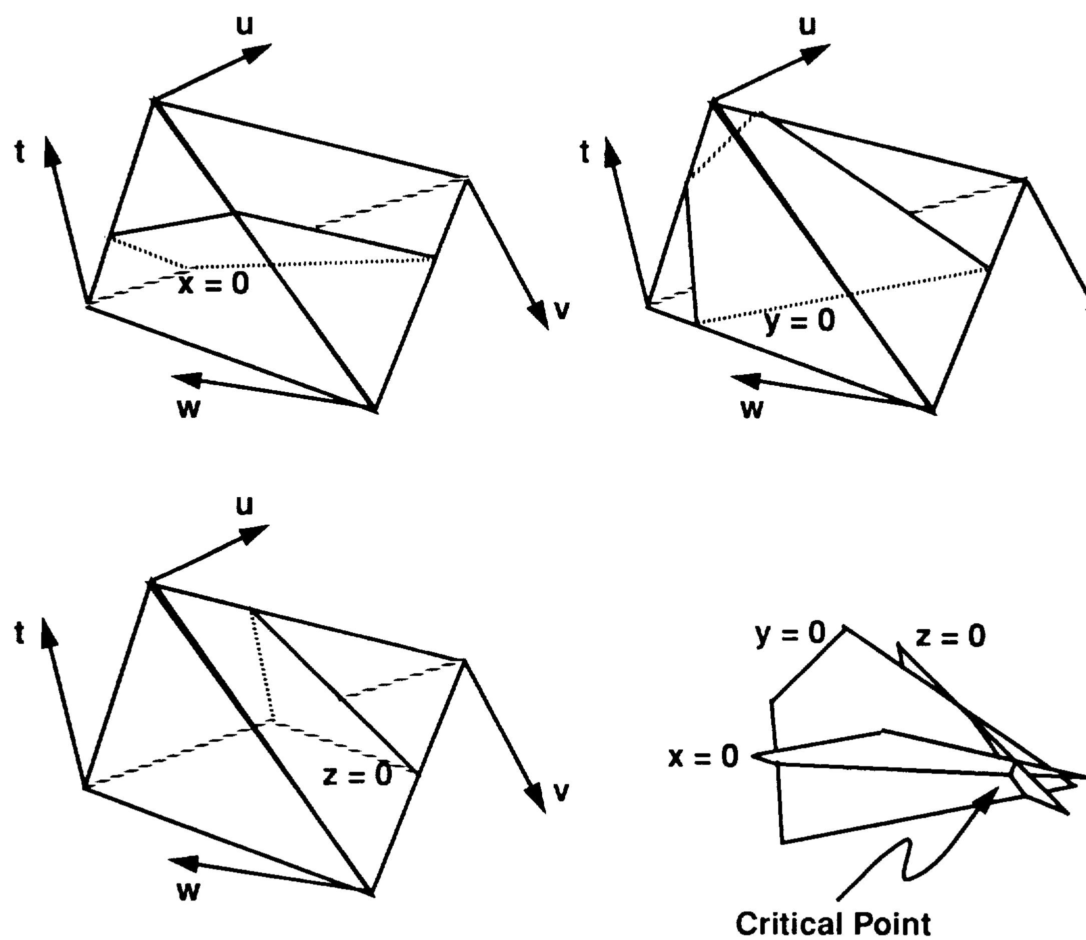
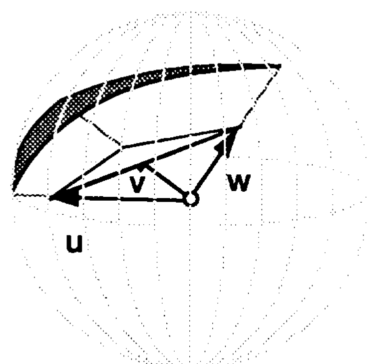
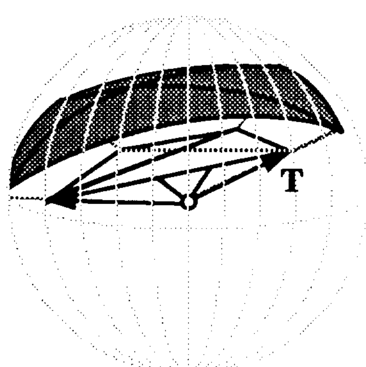


Figure 14: Locating a critical point in three dimensions.

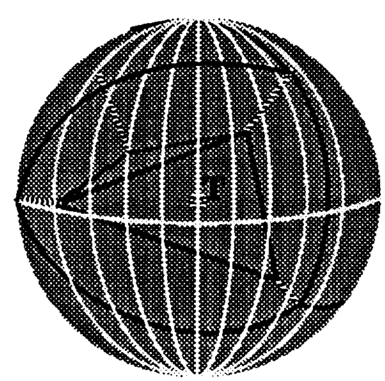
It was shown that for a three-dimensional vector to be between three other three-dimensional vectors it must intersect a triangle formed by the heads of the three vectors transposed to a common tail (figure 10). If a sphere represents all possible three-dimensional directions, then the gray patch in figure 15a represents all of the directions between **u**, **v**, and **w**. The directions of a tetrahedral cell are mapped to four overlapping patches on the sphere. These four patches form a double-covered four-sided patch, such that, except for boundaries each point on the four sided patch is in two of the triangular patches (figure 15b). The heads of the four vectors of the tetrahedral cell transposed to a common tail form a small tetrahedra (call it **T**). The intersection of a direction vector transposed to the common tail and **T**, corresponds to the isodirection line intersection with the tetrahedral cell. For a configuration such as in figure 15b, any direction will intersect **T** in two places or not at all. This means that if all four vectors lie in the same hemisphere, the isodirection line passes through the cell.



a: Directions between **u**, **v** and **w** mapped to a sphere.



b: **T** does not contain common tail.
No critical point is present.



c: **T** contains common tail.
There is a critical point in the cell.

Figure 15: Test for critical point in cell.

A special case is when **T** contains the common tail or equivalently, when the four direction vectors do not lie in the same hemisphere (figure 15c). For this special case an arbitrary direction vector will intersect **T** in one and only one location since **T** spans all directions. This special case is indicative of the tetrahedral cell containing a critical point, assuming monotonic interpolation schemes are applied for values in the cell interior. A simple test to determine whether the four vectors of a cell lie in a hemisphere also determines the presence of a critical point.

7. Results

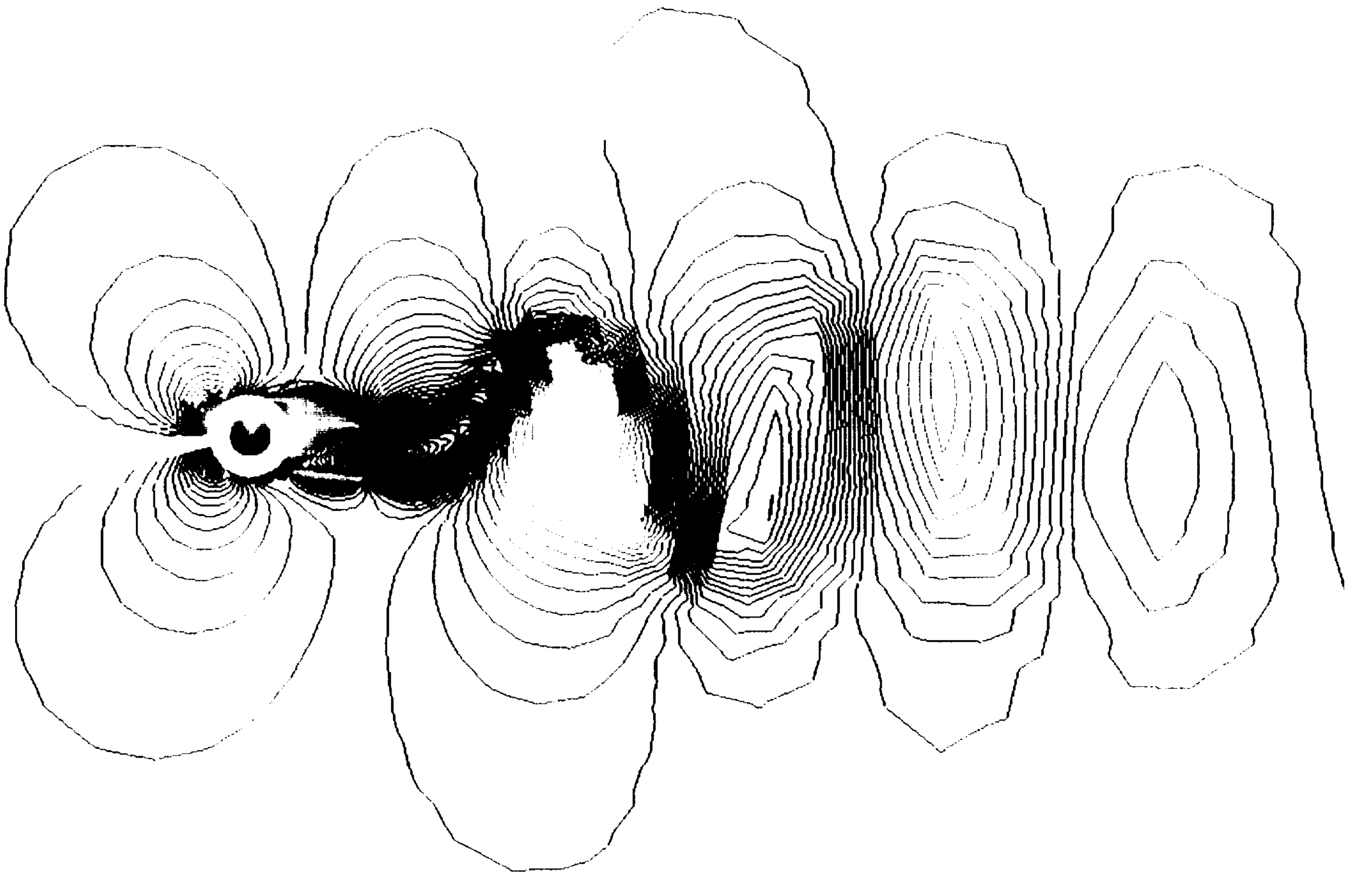
Several CFD flow solution data sets have been examined using isodirection lines, including flow past a tapered cylinder [10], a hemisphere cylinder [24], a NASA space shuttle orbiter [17], and a shuttle engine liquid oxygen posts [18].

The tapered cylinder data has been used to study unsteady flow in three-dimensions. The visualization of unsteady flow using isodirection lines shows the changes in the velocity vector field over time. Figure 16a shows isodirection lines for 128 directions for the velocity vector field of a two-dimensional cross section of the three dimensional data. Figure 16b shows a time sequence for a small subset of the two-dimensional cross section (the subset was also used in figure 6). A spiral attracting focus (vortex) and a saddle critical point are shown to be at opposite ends of several isodirection lines. The direction in red is the free stream direction. The blue and yellow isodirection lines delineate the back flow region. As the sequence proceeds the two critical points move downstream and the back flow region recedes. Finally the critical points merge and annihilate each other as the downstream-directed (vectors with positive x components) isodirection line segments (red, orange and violet) connect. This example highlights the ability of isodirection lines for two-dimensional vector fields to delineate regions where the vectors have a common direction characteristic.

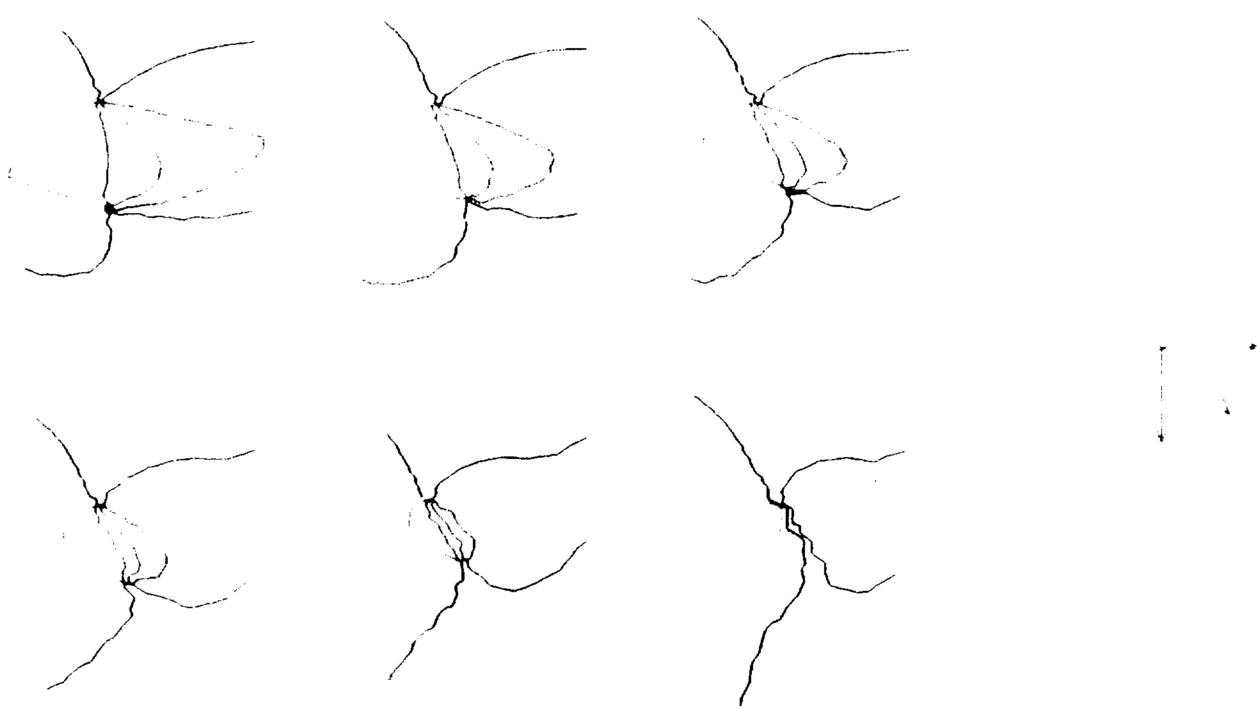
The isodirection line construction and critical point location algorithms for three-dimensions described above have been implemented and added to a developing visualization system using SuperGlue. SuperGlue [9] is a Scheme (a dialect of LISP) interpreter and pre-defined class hierarchy, which serves as a platform for the development of CFD visualization applications. This system was used to produce figures 17 and 18.

Figure 17 shows three views of thirty-two isodirection lines of the velocity vector field for flow past a hemisphere cylinder. The directions of the isodirection lines are color coded to the directions represented in the small icons (the directions are chosen in to form a cone with the axis of the cone equal to the free stream direction). This collection of vectors circumscribes a region similar to what was done in the two-dimensional example above. A more effective visualization would be to construct a surface from collections of isodirection lines.

The topology of flow past a hemisphere cylinder has been studied by several groups [4, 7, 24]. The critical point positions located using the algorithm in described in section 6 have been found to be in general agreement with those found using TOPO [4]. Differences can be attributed to the difference in numerical methods employed.



a: Isodirection lines for 128 directions.



b: Isodirection line time sequence (eight directions).

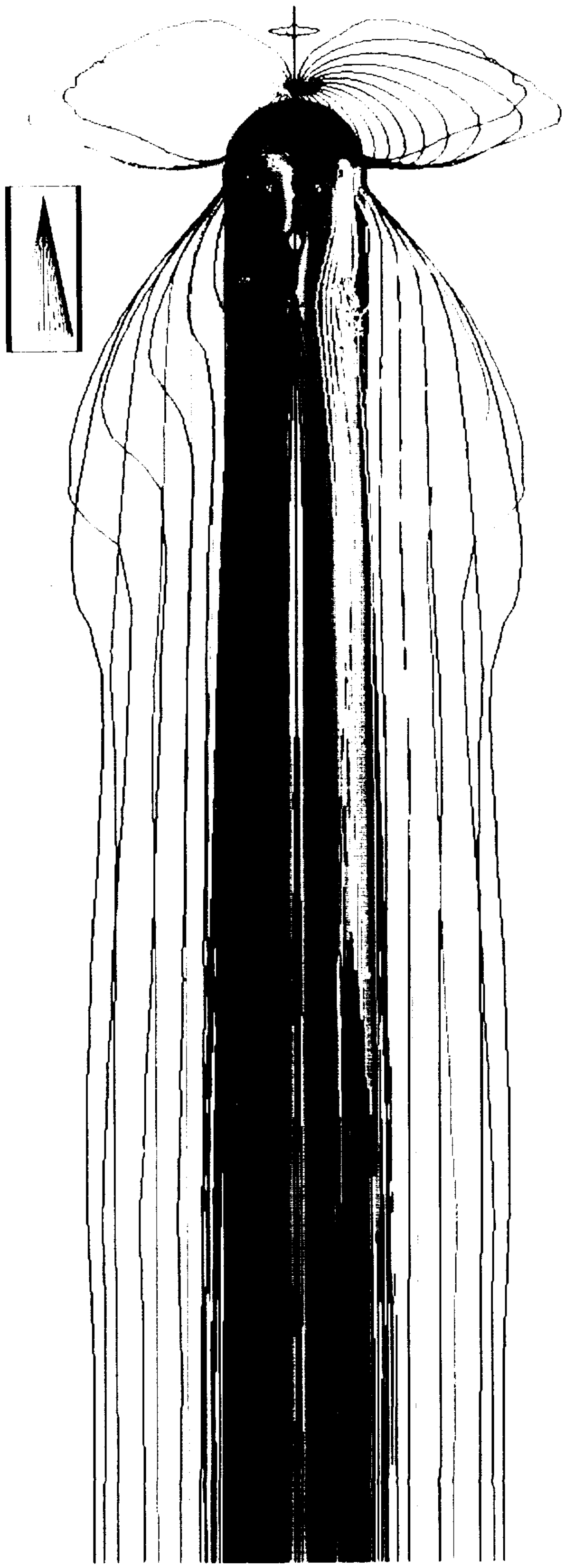
Figure 16: Two-dimensional flow about a cylinder.

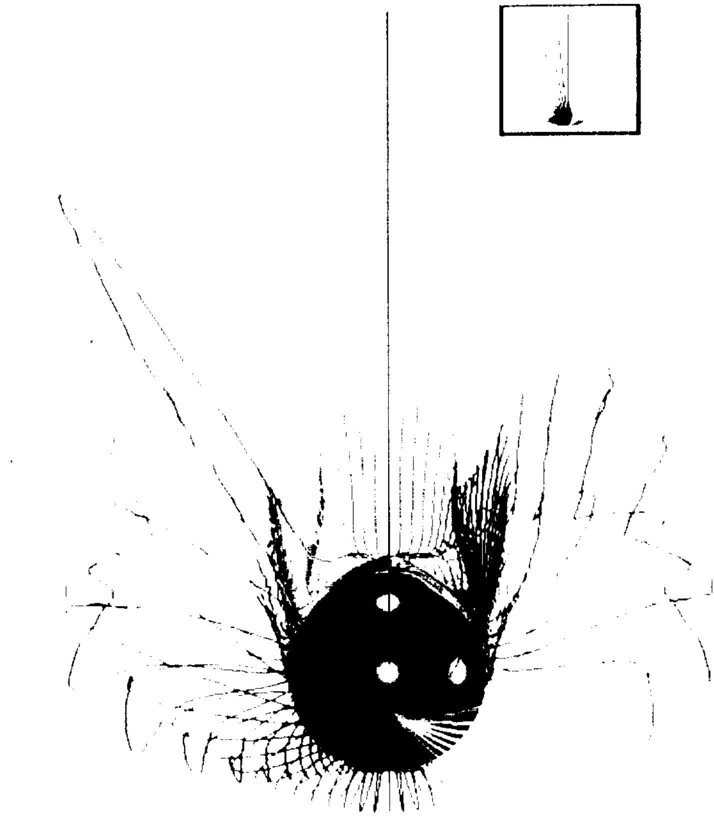
Following Page:

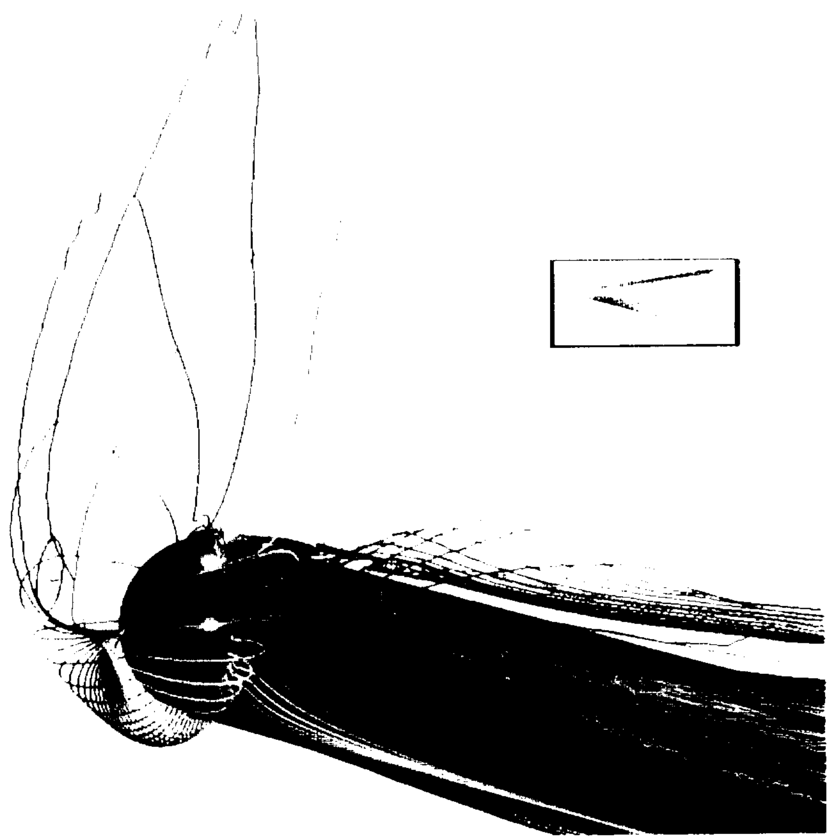
Figure 17: Three-dimensional flow past a hemisphere cylinder.

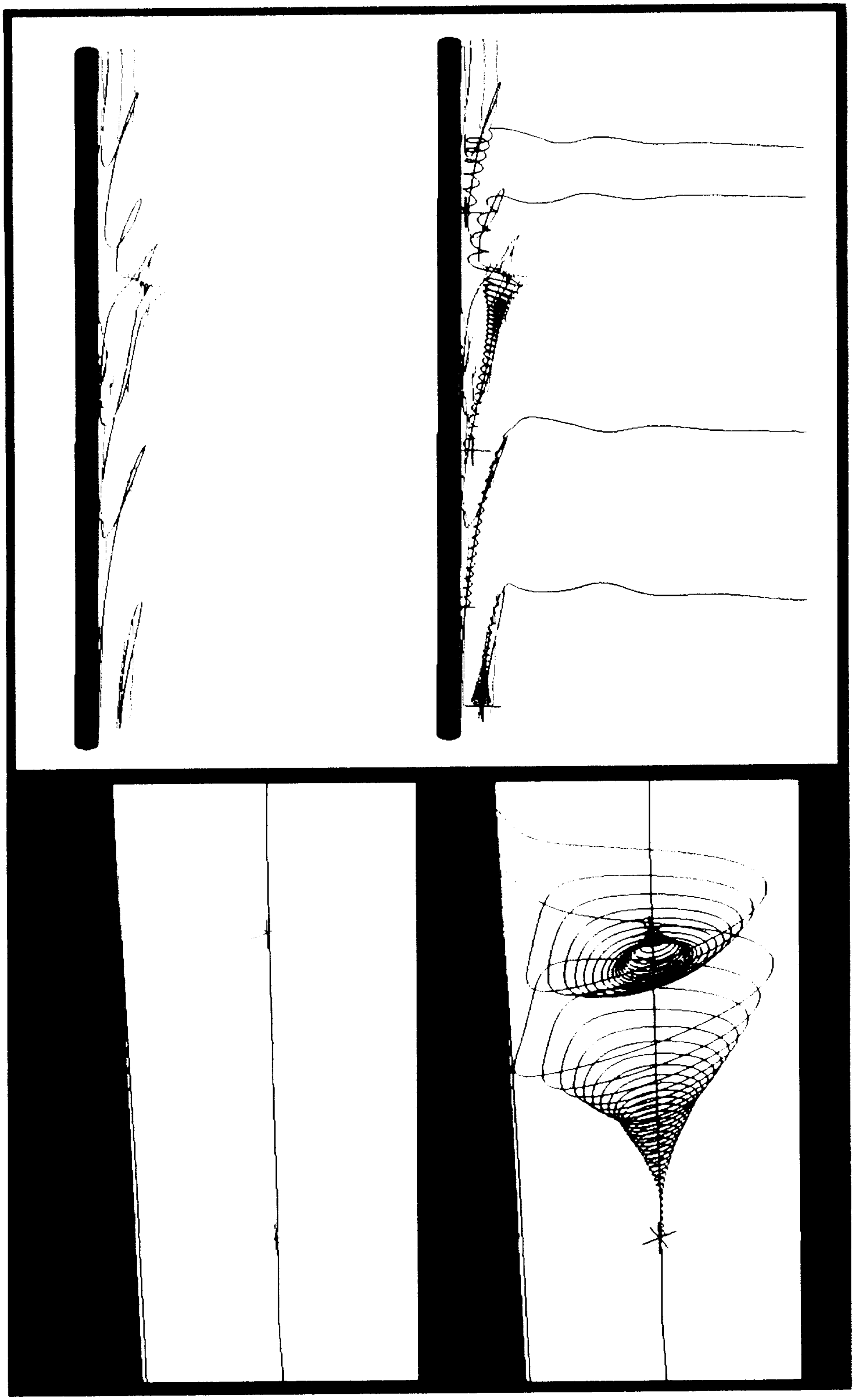
Second Following Page:

Figure 18: Three-dimensional flow past a tapered cylinder.









Integral curves initiated near critical points and isodirection line groupings can be used to reinforce the visual image of the flow dynamics. Figure 18 shows isodirection lines and integral curves depicting the topology of the velocity vector field for flow past a tapered cylinder. The image in the top left of figure 18 shows the isodirection lines for four orthogonal cross flow directions. The image in the top left of figure 18 shows several integral curves placed near interesting features delineated by the isodirection lines. The two images at the bottom of figure 18 show two isodirection lines intersecting at a critical point (the transition from red to blue in the bottom left image). Two integral curves with initial points at the cross-hairs are drawn showing the complex structure of the flow near the critical point. This example illustrates the capability of isodirection lines and critical points to guide the placement of the initial points of integral curves to portray interesting characteristics of a vector field.

8. Conclusions

This paper describes algorithms to construct isodirection lines for vector fields sampled over two- and three-dimensional curvilinear grids. This paper also describes algorithms for finding the critical points of these vector fields.

The goal of visualization is to use the pattern recognition capabilities of the visual sense to extract meaning from complex data. Visualization techniques have been applied to understand the topology of vector fields with varying degrees of success. Isodirection lines, critical points and integral curves are complementary visualization techniques, which can be used to effectively characterize interesting features of vector fields.

9. Acknowledgments

The author would like to thank Dan Asimov, Eric Barszcz, Jeff Hultquist, Tom Lasinski, Creon Levit, Al Globus, and Eric Raible for their technical advice. The author would also like to thank E. Lisette Gerald-Yamasaki, Jeff Hultquist, and Eric Raible for reviewing early versions of this paper and suggesting numerous improvements to the final presentation.

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